Partial Differential Equations Homework
(You don't have to turn this in, but you're responsible for the material on the final.)

1. Find the solution to the PDE $u_t = 3u_{xx}$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 4\sin 2x$.

2. Consider the wave equation $u_{tt} = c^2 u_{xx}$, where c is a constant. This equation governs the behavior of a vibrating string ($u(x,t)$ is the vertical displacement of the string at the point x at the time t). Let's assume that our string has length L and that its ends are fixed (so we have the boundary conditions $u(0,t) = u(L,t) = 0$ for all t.) (There's more background information on the wave equation at `http://www.math.duke.edu/education/ccp/materials/engin/wave/wave1.html` and `http://hyperphysics.phy-astr.gsu.edu/hbase/waves/waveq.html` . You can follow the links from the applets section of the course web page if you don't want to type in the addresses.)
   (a) Separate variables to find the general solution.
   (b) Find the solution to the PDE $u_{tt} = u_{xx}$, $u(0,t) = u(1,t) = 0$, $u(x,0) = \frac{1}{10}\sin \pi x - \frac{1}{20}\sin 3\pi x$, $u_t(x,0) = 0$.

ANSWERS

1. $4e^{-12t}\sin 2x$

2. (a) $u(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos \frac{n\pi ct}{L} + B_n \sin \frac{n\pi ct}{L} \right) \sin \frac{n\pi x}{L}$

   (b) $\frac{1}{10}\cos \pi t \sin \pi x - \frac{1}{20}\cos 3\pi t \sin 3\pi x$