Math 30(2) March 4, 2003 Name:_____

FIRST MIDTERM

This exam is 7 pages long; check that you have all the pages. Show your work. Correct answers with no justification may receive little or no credit. No calculators, notes, or books are allowed. No unnecessary simplification is required. Use the backs of pages if you run out of space, and make sure that I can find your answers.

THINK JOYFULLY

PROBLEM	POINTS	SCORE
1	15	
2	8	
3	15	
4	12	
5	20	
6	15	
7	15	
TOTAL	100	

(1) (15 pts) Consider the initial value problem $x' = y/x$, $y' = 2xy$, $x(0) = 1$, $y(0) = -4$. Use Euler's method with step size $h = 0.5$ to approximate $x(1)$ and $y(1)$. Is your answer likely to be nearly correct? Explain.

(2) (8 pts) Let $x(t)$ be Country 1's annual expenditure on arms, and $y(t)$ Country 2's. One of the systems below models the arms race between two countries that have historically gotten along pretty well, and the other system models the race between two countries that have fought a lot in the past. Which is which, and how do you know?

a) $\begin{aligned} x' &= 4y - x - 2 \\ y' &= 3x - 2y - 3 \end{aligned}$

b) $\begin{aligned} x' &= 3y - x + 1 \\ y' &= 4x - 2y + 2 \end{aligned}$

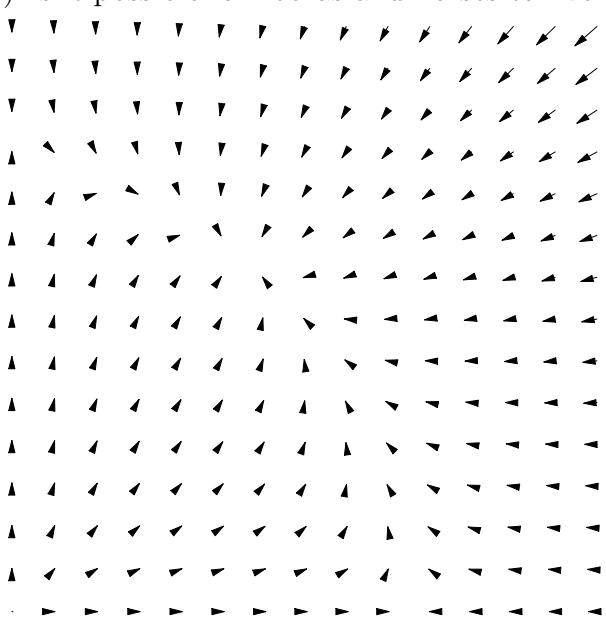
(3) (15 pts) Let $x(t)$ and $y(t)$ be the populations (measured in 1000s) of zebras and horses, respectively, two species that compete with each other for delicious grass. A model for their behavior is

$$\begin{aligned} x' &= 14x - 2x^2 - xy \\ y' &= 16y - 2y^2 - xy \end{aligned}.$$

(a) Find the four equilibrium points for the system. Do they all make physical sense?

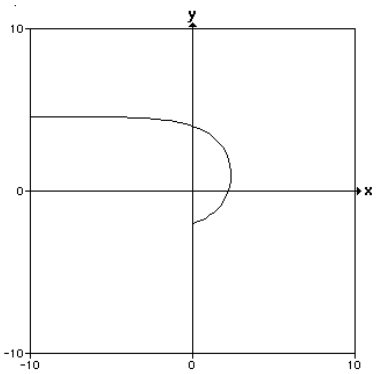
(b) The figure below shows the vector field for the system (the lower left corner is the origin of the xy-plane). What are the possible long-term behaviors of solutions?

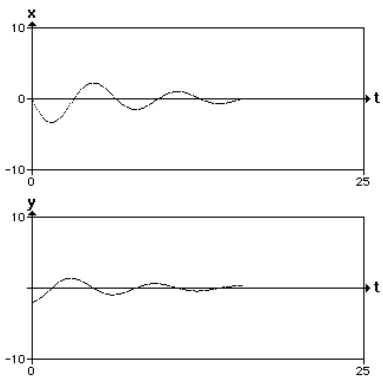
(c) Is it possible for zebras and horses to live together in loving harmony? Explain.



(4) (12 pts) For each of the following three systems, a solution is pictured in the phase plane and as separate $x(t)$ and $y(t)$ graphs. Identify which phase plane solution and which set of x and y graphs goes with each system.

System	Phase Plane Solution	x and y Graphs
$x' = 1 - y$, $y' = e^{x/2}$	_____	_____
$x' = -y^2$, $y' = 3\cos x$	_____	_____
$x' = 2y$, $y' = -x/2 - y/4$	_____	_____

A.


I.


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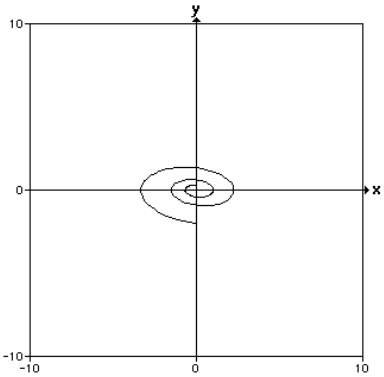

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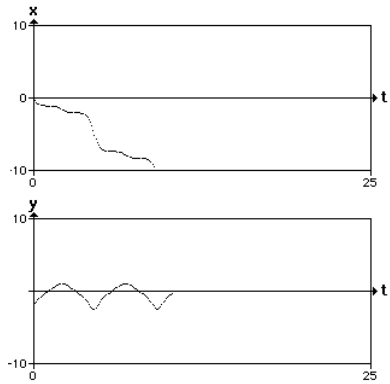
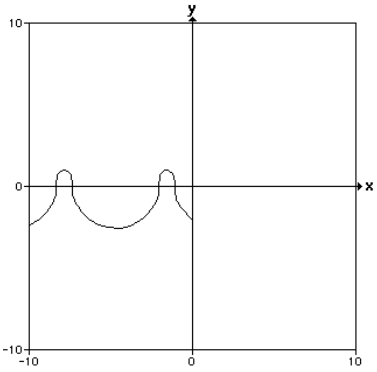
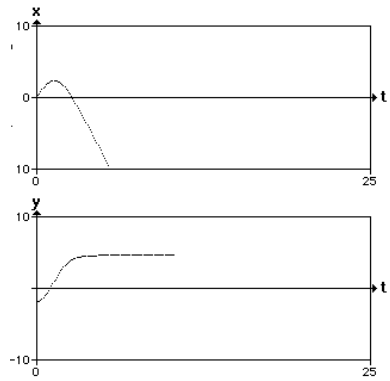


C.


III.


(5) (20 pts) Monkey Town initially has 1500 residents: 600 green monkeys and 900 purple monkeys. Suppose that 100 green monkeys and 100 purple monkeys move in each month. Assume further that each month 100 monkeys move out, with the number of each color moving out proportional to their share of the total population of Monkey Town. When the population reaches 3000, how many purple monkeys live in Monkey Town?

(6) (15 pts) Consider the family of differential equations $y' = y^3 - ay$, where a is a parameter.

(a) What is the bifurcation value for a?

(b) Draw phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation value. In each case, what are the possible long-term behaviors?

(c) If $y(t)$ is the cockroach population in Sharples, what do you hope is true about a?

(7) (15 pts) Find the general solution to the differential equation $y' + y^2 \sin t = 0$.