

MATH 119 PRACTICE FIRST MIDTERM

The actual midterm will of course be shorter than this. It will contain at least one question taken directly off your homework. You may use a calculator, but **not any of its calculus functions**. Don't expect too many "compute the integral" problems; that material is covered by the online Integral Proficiency Test.

- Chapter 6 Review (p. 306): #51, 55, 69. Chapter 7 Review (p. 361): #1-132 as needed, 156, 161.

Answers to odd questions are in the back of the book. #156: RIGHT(5), RIGHT(10), TRAP(10), Exact value, MID(10), LEFT(10), LEFT(5).

- For what type of integrand do the Left Sum and Right Sum approximations give the exact value for the integral on any interval? For what type of integrand do the Trapezoid and Midpoint approximations give the exact value for the integral on any interval?

The Left and Right Sums give the exact answer if the integrand is a constant; T & M give the exact answer if it's a straight line.

- Give an example of a function $f(t)$ such that $\lim_{t \rightarrow \infty} f(t) = 0$ but $\int_1^\infty f(t) dt$ diverges.

Lots of choices. For example: $\frac{1}{t}, \frac{1}{\sqrt{t}}, \frac{\sin t}{t}, \frac{1}{\ln t}, \frac{1}{t^3 + 7}, \dots$

- Assuming that the 440 feet is accurate and you neglect air resistance, determine the accuracy of the following paragraph. (A mile is 5280 feet.)

MY JOURNEY BENEATH THE EARTH

Condensed from "A Wolverine is Eating My Leg," by Tim Cahill

I am in Ellison's Cave, about to rappel down Incredible Pit, the second-deepest cave pit in the continental United States. The drop is 440 feet, about what you'd experience from the top of a 40-story building. If you took the shaft in a free fall, you'd accelerate to more than 100 miles an hour and then – about five seconds into the experience – you'd decelerate to zero. And die.

Yup, it's accurate.

- A rectangular lake is 100 km long and 60 km wide. The depth of the water at any point of the surface is a tenth of the distance from the *nearest* shoreline. How much water does the lake contain (in km^3)?

The maximum depth is $30/10 = 3$ km. The approximate volume of a horizontal slice is $(40 + 20h)(20h)\Delta h$ (see figure), so the volume of water is given by

$$\int_0^3 (40 + 20h)(20h) dh,$$

which turns out to be 7200.

