Calculus projects

(mostly stolen from David Richeson, Dickinson College)

(1) (History) Biographical sketch of Newton's life and Newton's contribution to the Calculus. You have a lot of freedom in this project. Your paper should include each of the following.
  (a) Give biographical information about Newton's life such as when and where he lived, what he was like as a person, etc.
  (b) Give information about his major scientific accomplishments.
  (c) Discuss his role in the development of calculus. Be as concrete as possible.
  (d) Compare Newton's view of calculus to our modern view.
  (e) You may want to discuss the Newton-Leibniz controversy over the invention of calculus.

(2) (History) Biographical sketch of Leibniz's life and Leibniz's contribution to the Calculus. You have a lot of freedom in this project. Your paper should include each of the following.
  (a) Give biographical information about Leibniz's life such as when and where he lived, what he was like as a person, etc.
  (b) Give information about his major academic interests and accomplishments.
  (c) Discuss his role in the development of calculus. Be as concrete as possible.
  (d) Compare Leibniz's view of calculus to our modern view.
  (e) You may want to discuss the Newton-Leibniz controversy over the invention of calculus.

(3) (History) Newton and Leibniz calculus controversy. You have a lot of freedom in this paper. Here are some topics you should include.
  (a) Discuss Newton's contribution to calculus.
  (b) Discuss Leibniz's contribution to calculus.
  (c) Discuss how their view of the calculus differs from today's view.
  (d) Discuss the controversy over the invention of calculus.
  (e) What was the opinion at the time? What is the opinion today?

(4) (History, physics) Where do functions come from? This project is based on Atkinson's article "Where do functions come from?" (College Math. Journal, 33 #2), but you'll need to get more information from other sources.
  (a) Discuss the history of the study of motion.
  (b) Discuss the origin of the notion of a function.
  (c) What is meant by "function" today? Does the idea of the derivative make sense for every function?

(5) (History) History of calculus notation. The notation that we use to express the ideas of calculus has not always been in existence. Someone needed to come up with the notation for a function, a derivative, an integral, etc. The article "Calculus Notation" found in *Readings for Calculus* gives a nice overview of the history of some of the notation used in calculus.
  (a) Give a brief history of the notation used in studying Calculus. You should discuss the history of the notation for the derivative, the integral, the function, the limit and any other interesting tidbits.
  (b) Translate some of our current formulas into "old" notation. For instance, you may want to translate the product rule, the quotient rule, the Fundamental

Theorem of Calculus, the mean value theorem, etc. You must do several examples not found in the text of the article.

(6) (History, related rates) Lengthening Shadow: the story of related rates. This project is based on the journal article *Lengthening Shadow: the story of related rates*, by Austin, Barry, and Berman (Math Mag., 2000).
   (a) Summarize the article discussing the history of related rates problems.
   (b) Discuss the purpose of introducing related rates problems to a calculus course.
   (c) Discuss the current movement away from related rates problems.
   (d) You may want to interview the mathematics professors here to determine their opinion of related rates problems.
   (e) Find copies of old textbooks and give examples of the first related rates problems.
   (f) Solve these problems.

(7) (Mean value theorem) On a mean value theorem. This project is based on Mercer's article "On a mean value theorem" (College Math. Journal, 33 #1). The article discusses a new version of the Mean Value Theorem.
   (a) The argument given is short and sketchy. Rewrite it, including all the details, so that is is easy to follow.
   (b) How is this theorem similar to the usual Mean Value Theorem? How is it different?
   (c) Can you give a practical application of this new theorem?
   (d) What are some other versions of mean value theorems?

(8) (Geometry) Tangents without calculus. (This project is more challenging mathematically than most of the others.) It's based on Aarao's article "Tangents without calculus" (College Math. Journal, Nov. 2000), which talks about finding tangent lines to polynomials without using limits.
   (a) The argument given is short and sketchy. Rewrite it, including all the details, so that is is easy to follow.
   (b) Generalize the results to "tangent parabolas." (The notion of a second-degree Taylor polynomial may be helpful here.)

(9) (Related rates) The falling ladder paradox. The article *The falling ladder paradox*, by Paul Scholten and Andrew Simoson (College Math. Journal, Jan. 1996) investigates the classic falling ladder problem and the paradoxical situation that seem inevitable.
   (a) State and solve the classic falling ladder problem (you may assume that the ladder is 40 ft long and that the base is moving at a rate of 5ft/sec).
   (b) What is the paradox?
   (c) Find the location at which the top of the ladder is traveling at light speed.
   (d) Use the methods in the paper to find when the ladder separates from the wall. You will want to go into more detail than the author does (since he doesn't show all of his work).
   (e) Repeat the analysis for a massless ladder with a person of mass m standing on the top. When does the ladder leave the wall?

(10) (Physics) How not to land at Lake Tahoe. The article *How Not to Land at Lake Tahoe!*, by Richard Barshinger (Amer. Math. Monthly, May 1992) creates a model of a descending airplane for various situations. He is very sketchy with the details

of his argument. Rewrite his analysis including all of the details so that the argument easy to follow.

(11) (Physics, optimization) Refraction and reflection of light: Snell's law
  (a) What is Fermat's principle of optics?
  (b) Suppose a light is shined from point P to point Q by reflecting it off of a mirror. P is located a units from the mirror and Q is located b units from the mirror, the horizontal distance between P and Q is L. Find the point where the light reflects off the mirror. Show that the angle of incidence equals the angle of reflection.
  (c) Give an interesting example with actual numbers for a, b and L.
  (d) State and prove Snell's law. In particular, suppose a light is shined from point P to point Q where P is located in the air a units above the pool of water (or any fluid), Q is located b units below the surface of the water, and the horizontal distance between P and Q is L. Assume that v_1 is the velocity of the light in air, v_2 is the velocity of light in water and that the beam of light makes angles θ_1 and θ_2 with the vertical (y-axis) in the air and water respectively. Find a relationship between v_1, v_2, θ_1, and θ_2.
  (e) Be sure to define the index of refraction and the refraction angle in your discussion.
  (f) Give an interesting example with actual numbers for a, b and L.

(12) (Physics, optimization) The calculus of rainbows. For this work you may wish to consult the article *Somewhere within the rainbow* by Steven Janke and *The Calculus of Rainbows* by Rachel Hall and Nigel Higson.
  (a) State Fermat's principle of optics.
  (b) State Snell's law and indicate how you would use calculus to prove Snell's law (you don't have to prove it).
  (c) Explain the behavior of sunlight in a raindrop.
  (d) Suppose the sun is in the sky making an angle θ with the horizon. You stand with your back to the sun. At what angle should you gaze to find the rainbow?
  (e) Specifically, what if the sun is setting ($\theta = 0$)? What if the $\theta = 45°$?
  (f) Discuss (but don't prove) why the rainbow appears as a band of colors in the sky.
  (g) Discuss (but don't prove) why we frequently see a second, lighter rainbow above the main rainbow.

(13) (Optimization) Do dogs know calculus? For this project you may wish to consult the article *Do dogs know calculus?* by Timothy Pennings (College Math. Journal, May 2003).
  (a) Suppose a dog can run with a velocity v_l on the land and can swim with a velocity of v_w. You and the dog are standing on the shore of a lake and you throw a ball in the water. The ball lands a feet off shore and b units to your right. What route should the dog take to reach the ball fastest?
  (b) Pick reasonable values for v_l, v_w, a and b and compute the time for this optimal path. Also, compute the time for the direct (swimming only) route and the "right angle" route.
  (c) Set up your own test case. Here's an idea. Have a starting line in the grass and a finish line on a sidewalk. Have the runner (a person, probably) start

carrying a heavy object. The person can drop the object when (s)he reaches the sidewalk.

(i) Test the person's running speed with and without the weight.
(ii) Run numerous trials with different routes.
(iii) Use calculus to compute the best route.
(iv) Compare your theoretical and experimental results.

(14) (Optimization) A new wrinkle on an old folding problem. This project is based on the article *A new wrinkle on an old folding problem* by Greg Frederickson (College Math. Journal, 2003).

(a) State and prove the classic "box folding" problem. That is, given a piece of paper of length l and width w, find the largest box that can be made by cutting squares out of the corners.
(b) Discuss the history of this problem.
(c) Reproduce the results of the paper. The author is very sketchy with the details of his argument. Rewrite his analysis including all of the details so that the argument easy to follow.
(d) Include a template for the cistern for an 8.5"$\times$11" piece of paper.

(15) (History) History of calculus in Egypt, Greece, and India. It's generally agreed that Newton and Leibniz invented calculus, but other people came close, some of them long before and far away from those two.

(a) Give a brief history of Egyptian (e.g., the Moscow mathematical papyrus, Alhazen), Greek (e.g., Eudoxus, Archimedes), and Indian (e.g, Narayana et al.) contributions to calculus.
(b) In what sense is it true that each of these groups "knew calculus"?
(c) In what sense is it false?

(Some possible starting points for this project are Joseph's book *The Crest of the Peacock* and several articles by David Bressoud. Some history of math or history of calculus books have a little information on the subject.)

(16) (History) Hyperbolic functions and their history. We skipped the section on the hyperbolic functions (3.8 in Hughes-Hallett et al.). In this project, you'll fill us in.

(a) Define the hyperbolic functions and discuss their properties, practical uses, and derivatives.
(b) How are they like the trig functions? How do they differ?
(c) Discuss their origin and history.

(One possible source is Barnett's article "Enter, Stage Center: The Early Drama of the Hyperbolic Functions" (Math. Magazine, 77 #1).)

(17) (History) Women in calculus. A great place to start on this project is Dr. Riddle's website, Biographies of Women Mathematicians, http://www.agnesscott.edu/lriddle/women/women.htm .

(a) Discuss some of the contributions that women have made to the development of calculus.
(b) What factors kept women from contributing more? (Why are almost none of the results in your textbook named after women?)
(c) To what extent are these factors still in existence? (You may want to interview students and faculty about their experiences.)