

MATH 118 PROJECT #1  
Due Wednesday, February 27

Work in groups of three to five people. Turn in a report of your results (with complete sentences, please, and graphs and figures as appropriate) in class on Wednesday, February 27 – one report per group, pledged by all of the group members. Think of a classmate who hasn't thought about these questions as the target audience for your report. *You may not discuss your work with members of any other group.*

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For this project we will look at two sets of data, one for U.S. population and one for U.S. yearly per capita energy consumption. If  $P(t)$  is the population and  $E(t)$  the per capita energy consumption, then their product  $T(t) = P(t) \cdot E(t)$  is the *total* yearly energy consumption in the U.S. We will consider what these functions and their derivatives tell us, and how the derivative for  $T(t)$  relates to the derivatives of  $P(t)$  and  $E(t)$ .

Here are the data. The population is measured mid-year, in millions; yearly per capita energy consumption is measured in  $10^8$  BTU.

Year	Population $P(t)$	Per Capita Energy Use $E(t)$
1982	232.2	3.05
1983	234.3	3.01
1984	236.4	3.13
1985	238.5	3.10
1986	240.7	3.08
1987	242.8	3.16
1988	245.1	3.27
1989	247.3	3.29

- Estimate the rates of change of population and per capita energy use in 1989. (That is, estimate  $P'(1989)$  and  $E'(1989)$ .)
  - Use these data to estimate the population in 1990. What was the population really? What might explain the difference between your estimate and the real population?
  - Now use these data to estimate the population in 2000. What was the population really? What might explain why your estimate for 2000 was so much worse than your estimate for 1990?
- For  $t = 1983, 1984,$  and  $1985$ , compute  $T(t)$  and estimate  $P'(t), E'(t),$  and  $T'(t)$ .
  - We might think that because  $T$  is the product of  $P$  and  $E$ , its derivative is the product of  $P'$  and  $E'$ , that is, that  $T'(t) = P'(t) \cdot E'(t)$ . Does this seem to be true? What's going on?
  - We could also estimate the derivative of the sum  $P(t) + E(t)$ . Why aren't we doing that?
- Usually we think of the U.S. population  $P(t)$  as a smooth function of time. To what extent is this justified? What happens if we zoom in at a point on the graph? What about events such as the Louisiana Purchase? Or the moment of a baby's birth?
  - What do we in fact mean by the rate of change of the population at a particular time  $t$ ?
  - Give another example of a real-world function which is not smooth but is usually treated as such.