Math 115 Practice Second Midterm

1. High levels of cholesterol in the blood are not healthy in either humans or dogs. A clinic compared healthy dogs it owned with healthy pets brought to the clinic to be neutered. The summary statistics for blood cholesterol levels (milligrams per deciliter of blood) appear below (results published in the *American Journal of Veterinary Research*).

Group	n	**average**	s
Pets	26	193	68
Clinic	23	174	44

(a) State null and alternative hypotheses to test for any difference between clinic dogs and dogs owned as pets. Be sure to define any symbols that you use.
(b) Compute a test statistic and P-value for this test. What is your conclusion?
(c) Compute a 95% confidence interval for the difference between clinic dogs and dogs owned as pets, and explain what this interval represents.

(a) Let μ_1 be the population mean cholesterol for pet dogs, and μ_2 the population mean cholesterol for clinic dogs. $H_0 : \mu_1 - \mu_2 = 0$ (no difference). $H_a : \mu_1 - \mu_2 \neq 0$ (there is a difference) (two-sided).

(b) This is a two-sample t-test (comparing means). $t = \dfrac{(193-174)-0}{\sqrt{\frac{68^2}{26}+\frac{44^2}{23}}} = 1.17$ with df = 23-1=22. By Table D, $P(T \geq 1.17)$ is between .10 and .15. So, for the two-sided alternative, the P-value is between .20 and .30. So we do not reject H_0: even though the pets had slightly higher cholesterol than the clinic-owned dogs, this could easily be due to chance, and we can't be sure that the two groups don't have the same average cholesterol.

(c) 95% CI: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 19 \pm (2.074)(16.19) = 19 \pm 33.58 = (-14.58, 52.58)$. This is a range of values for the true difference in cholesterol levels. 95% of samples like ours will have the true value in the CI, so we're 95% confident that the true difference lies in this interval.

2. To play a game, you must pay \$5 for each play. There is a 10% chance that you will win \$5, a 40% chance that you will win \$7, and a 50% chance that you will win only \$3.
(a) What are the mean and standard deviation of your *net* winnings?
(b) You play twice. Assuming that the plays are independent events, what are the mean and standard deviation of your total net winnings?
(c) You play three times. What is the probability that you win \$7 all three times? What is the probability that you don't win \$7 on any of your three tries?

(a) Your net winnings are your winnings minus the \$5 it costs you to play. Let the random variable X be your net winnings. The distribution for X is

X	**0**	**2**	**-2**
Prob	**.1**	**.4**	**.5**

The mean of X is $\mu = 0(.1) + 2(.4) + (-2)(.5) = -.2$. The standard deviation is $\sigma = \sqrt{(0-(-.2))^2(.1) + (2-(-.2))^2(.4) + (-2-(-.2))^2(.5)} = 1.89$.

(b) Let X_1 be your net winnings on the first play, and X_2 your net winnings on the second play. Then your total net winnings are $X_1 + X_2$. Means add, so $\mu_{X_1+X_2} = \mu_{X_1} + \mu_{X_2} = (-0.2) + (-0.2) = -0.4$. Since the variables are independent, variances add, so $\sigma^2_{X_1+X_2} = \sigma^2_{X_1} + \sigma^2_{X_2} = (1.89)^2 + (1.89)^2 = 7.12$, and $\sigma_{X_1+X_2} = \sqrt{7.12} = 2.67$.

(c) Prob(win \$7 all three times) = (.4)(.4)(.4)=.064. The probability of not winning \$7 on a particular play is 1-.4=.6, so Prob(don't win \$7 any of the three plays) = (.6)(.6)(.6)=.216.

3. People with O-negative blood are called "universal donors" because O-negative blood can be given to anyone, regardless of the recipient's blood type. Only 6% of people have O-negative blood.

(a) Suppose 20 people come to a blood drive. What are the mean and standard deviation of the number of universal donors among them? What is the probability that there are either 2 or 3 universal donors?

(b) The Tennessee Red Cross anticipates the need for at least 1850 units of O-negative blood this year. It estimates that it will collect blood from 32,000 donors. How great is the risk that the Tennessee Red Cross will fall short of meeting its need?

(c) Why is O-positive blood so much better than O-negative blood?

(a) Let the random variable X equal the number of universal donors at the blood drive. X is distributed approximately Binomial(20,.06), so $\bar{X} = np = (20)(.06) = 1.2$. The standard deviation is $\sqrt{np(1-p)} = \sqrt{(20)(.06)(.94)} \approx 1.06$. P($X = 2$ or 3)=P($X = 2$) + P($X = 3$) = (by table C) .2246+.0860=.3106.

(b) Use the normal approximation: Binomial(32000,.06) is approximately Normal($np, \sqrt{np(1-p)}$) = Normal($(32000)(.06), \sqrt{(32000)(.06)(.94)}$) = Normal(1920,42.48). So P($X < 1850$) $\approx$ P$\left(z < \frac{1850-1920}{42.48}\right) \approx P(z < -1.65) \approx .05$ (using table A). There is about a 5% chance that this Red Cross chapter will run short of O-negative blood.

4. The Pew Research Center reports that they are actually able to contact only 76% of the randomly selected households drawn for a telephone survey. Which of the following might be modeled by a Binomial random variable?

(a) The number of households Pew has to call before they successfully contact 800.

(b) The number of successful contacts from a list of 1000 sampled households.

(c) The average household income of the contacted household.

Only (b) is Binomial (a count of successes in a fixed number of trials).

5. A resident with a radar gun took a random sample of the speeds (in miles per hour) of 23 cars in his neighborhood. The average speed was 31.0 mph, with standard deviation 4.25 mph, and the data were roughly normal. Can we be 95% confident that the average speed of cars in the neighborhood is greater than the 30-mph speed limit?

Use a 1-sample t test. Let μ be the average speed. $H_0 : \mu = 30$. $H_a : \mu > 30$ (one-sided). The test statistic is $t = \frac{31-30}{4.25/\sqrt{23}} = 1.128$. We have 23-1=22 degrees of freedom, so table D gives us a P-value of between .10 and .15. Thus we cannot reject the null hypothesis at level $\alpha = .05$; i.e., we cannot be 95% confident that the average speed is greater than 30 mph.

6. For each of the situations described below,

- Would you use a one-sample t, a two-sample t, or a matched-pairs t (or none of these)?
- Would you perform a hypothesis test or find a confidence interval?

(a) Random samples of 50 men and 50 women are asked to imagine buying a birthday present for their best friend. We want to estimate the difference in how much they are willing to spend.

(b) Mothers of twins were surveyed and asked how often in the past month strangers had asked whether the twins were identical.

(c) Are parents equally strict with boys and girls? In a random sample of families, researchers asked a brother and sister from each family to rate how strict their parents were.

(d) Forty-eight overweight subjects are randomly assigned to either aerobic or stretching exercise programs. They are weighed at the beginning and at the end of the experiment to see how much weight they lost.

(i) We want to estimate the mean amount of weight lost by those doing aerobic exercise.

(ii) We want to know which program is more effective at reducing weight.

(e) Couples at a dance club were separated and each person was asked to rate the band. Do men or women like this group more?

(a) These are independent groups sampled at random, so use a two-sample t confidence interval to estimate the size of the difference.

(b) There is only one sample. Use a one-sample t-interval.

(c) Brother and sister form a matched pair. Use a matched-pairs t hypothesis test.

(d) (i) Before-and-after studies call for matched-pairs. Find a matched-pairs t confidence interval. (ii) The two treatments were assigned randomly, so use a two-sample t hypothesis test.

(e) Hard to say. Most likely, couples would discuss the band or even decide to go to the club because they both like a particular band. If we think that's likely, then these are matched pairs. But maybe not. If we asked them their opinion of, say, the bartender's hairstyle, the fact that they were a couple might not affect the independence of their answers, and we should use a two-sample t. Either way, we do a hypothesis test.

7. What is meant by the power of a test? A type I error? A type II error?

See section 6.4.

8. At Angus Scot College in Decatur, Scotland, each semester students in the intro stats class measure the lengths of their kilts as part of a class survey. There could be a systematic difference between the kilts worn on the first days of the fall semester and those worn on the first day of the spring semester (due to the weather, for example). Below is a table with the sample average and standard deviation of kilt lengths in inches for surveys done this fall and last spring:

Semester	n	Mean	Std. Dev.
Fall	98	21.44	2.42
Spring	101	22.40	2.18

(a) Consider these to be simple random samples of 2005-2006 fall and spring semester Angus students (and their kilts). Define parameters and carry out a test to see if there is significant evidence for a difference in mean kilt lengths for the fall and spring semesters. Report your P-value and conclusion.

(b) Which of the following statements gives the best interpretation of the P-value in part (a)? Choose one (i-v):_____

(i) It is the probability that the mean kilt length is the same for both semesters.

(ii) It is the probability that the mean kilt length is different for fall and spring.

(iii) It is the probability of seeing averages that differ by exactly 0.96 inches if the means are the same.

(iv) It is the probability of seeing averages that differ by 0.96 inches or more if the means are the same.

(v) It is the probability of seeing averages that differ by 0.96 inches or less if the means are the same.

(a) If we had measured the same students in each semester, we could have done a matched-pairs one-sample t-test. Since we didn't, we'll use a two-sample t-test. Let X_1 be the length of a random kilt in the fall semester (with mean μ_1), and X_2 the length of a random kilt in the spring semester (with mean μ_2). Then H_0: $\mu_1 = \mu_2$, and H_a: $\mu_1 \neq \mu_2$. Our test statistic is $t = \frac{22.4-21.44}{\sqrt{\frac{2.18^2}{101}+\frac{2.42^2}{98}}} \approx \frac{.96}{.3268} \approx 2.94$. Use df$= 98-1 = 97$. We skip down to df$= 80$ in Table D (we could probably also skip up to 100) to get a one-sided P-value of between .0025 and .001, so our two-sided P-value is between .005 and .002. Therefore we can reject H_0 at level $\alpha = .005$, and we conclude that there is strong evidence that kilt lengths are different in the fall and spring.

(b) (iv)

(c) In part a, we found the standard error to be .3268. The $t(80)$ 99% critical value is 2.639, so the CI is $.96 \pm (2.639)(.3268) = (0.097, 1.822)$. If we use 100 for df, the critical value is 2.626, and the CI is $.96 \pm (2.626)(.3268) = (0.102, 1.818)$.